where $\omega_{\vec{q},p}$ is now the angular frequency of the phonon with wave vector $\vec{q}$ and polarization p. We introduce the 1/N to cancel the N from the sum Eq. (IV-30).

Then

$$U_{k,k'} = \sum_{p=1,2,3} \frac{B^{1/2}}{\omega_{\vec{q},p}} \; JS(\theta)\, \hat{e}_{\vec{q},p} \cdot \hat{s} \qquad \text{(IV-35)}$$

and dropping the sum over p with the understanding that we will consider the polarization that gives the largest contribution to $U_{kk'}$ we obtain

$$p_{\vec{k},\phi}(\theta) = \frac{B}{4\pi^2 \hbar \, |\nabla_k E|_{k'}} \; \frac{[JS(\theta)]^2 (\hat{e}_{\vec{q},p} \cdot \hat{s})^2}{\omega^2_{\vec{q},p}} \; . \qquad \text{(IV-36)}$$

Lumping all the uninteresting constants together into D, we have

$$\frac{1}{\tau(\vec{k})} = D \int_0^{\pi} d\phi \int_0^{2\pi} d\theta \; \frac{(1-\cos\theta)\,|\sin\theta\,|\,[JS(\theta)]^2 (\hat{e}_{\vec{q},p} \cdot \hat{s})^2}{\omega^2_{\vec{q},p} \, |\nabla_k E|_{k'}} \; . \qquad \text{(IV-37)}$$

We now consider the sources of anisotropy in $\tau(\vec{k})$. The density of states factor $\frac{1}{|\nabla_k E|_{k'}}$ is, strictly speaking, isotropic since we assumed spherical constant energy surfaces to derive Eq. (IV-20). For a warped surface, one could assume Eq. (IV-20) was still valid and compute $\tau(k)$ using the density of states factor. An anisotropic state density will act as a weighting factor in the integral; for the case of an alkali we can estimate its magnitude using Eq. (IV-4).

$$|\nabla_k E| = 1/\left(\frac{\partial k}{\partial E}\right)_{E_F} = \frac{1}{k'_o [1 + B Y_4 + B_1 Y_6]} \; . \qquad \text{(IV-38)}$$

Equation (IV-10) gives B_1 in terms of A_1. For a typical value of A_1, -.02, B_1 = -.09 and noting the values of Y_6 in the principal directions we see that the density of states factor varies by about $\pm$ 10 percent from its average value. This is a relatively weak weighting factor, compared with the effect of $\omega^2_{\vec{q},p}$.

We now write

$$\omega_{\vec{q},p} = c_{\vec{q},p}\, q \qquad \text{(IV-39)}$$

and choose for $c_{\vec{q},p}$ the velocity of sound in the elastic limit (small q). While this is incorrect for large q, the differences in the elastic limit velocities for different polarizations and for different directions of propagation are quite large and this approximation indicates the relative importance of different branches of the phonon spectrum and of different directions of the same branch, even though the individual q's within a branch are not correctly weighted. We indicate the choice of the elastic limit sound velocity by writing $c_{\hat{q},p}$, where $\hat{q}$ is a unit vector in the direction of $\vec{q}$. In Table 4-3 we list expressions for the velocity of sound squared times the density for the three principal directions and evaluate the expressions using the elastic constants for potassium, as given in Table 4-1 [13]. The point is that $c^2_{\hat{q},p}$ varies by about a factor of six between the various transverse modes and is substantially larger for the longitudinal modes than for the transverse ones. This weights the contribution of the transverse phonons to the integral more strongly than that of the longitudinal ones. Furthermore, it means that transitions using certain phonons, namely 110 phonons polarized 110 will be weighted very much more strongly than others. Transitions from a given initial state $\vec{k}$ on the Fermi surface to those final states for which the transition probability is large, involve many different phonon directions and so average the different sound velocities to some extent. Since the angular terms in the integral weight certain values of the scattering angle, θ, heavily and since the phonon direction for a fixed scattering angle depends on the initial state $\vec{k}$ we do not expect the variations in sound velocity to average out completely, although we do expect that the averaging will make the anisotropy of $\tau(\vec{k})$ considerably less than that of $c^2_{\hat{q},p}$. The numerical values in Table 4-3 indicate how bad the assumption of an elastically isotropic solid, usually made in computing τ, is for the alkali metals.

The term $1/q^2$ in the integral for $1/\tau(\vec{k})$ depends on the initial state $\vec{k}$ for those processes in which $K \neq 0$ in Eq. (IV-30). For a normal process, in which $K = 0$, $\vec{q}$ depends only on the angle θ between k and k'. In an umklapp(U.K.) process, in which $K \neq 0$, $\vec{q}$ depends not only on θ, but also on